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Multi-Objective Stochastic Optimization for Preventive Maintenance Planning

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Abstract

Maintenance is an essential part of mechanical integrity programs and aims to prevent the occurrence of process safety incidents and costly unplanned shutdowns. Maintenance can increase the reliability of equipment in productive systems and effective preventive maintenance programs enable maintenance activities to be planned proactively. However, maintenance planning is subject to resource scarcity and is rendered nontrivial due to system complexity, reliability model nonlinearity, and parametric uncertainty.

Multi-objective stochastic mixed-integer nonlinear programming is well suited to addressing these challenges and is adopted here to optimize the time intervals in which to perform maintenance on different pieces of equipment. Following presentation of an optimal maintenance planning framework, a model is formulated and optimized accounting for: the effect of imperfect repair using an effective age model, equipment failure behavior using a Weibull reliability model, endogenous uncertainty in reliability model parameters, and the simultaneous need to satisfy the competing objectives of cost minimization and reliability maximization using the ε -constraint method. The results of the research consist of optimal maintenance plans, plots of resultant equipment and system reliability over time, and a Pareto frontier of optimal solutions from which the decision maker can select. The approach adopted here is illustrated with a case study and can be extended to improving the overall availability, effectiveness, and resilience of a variety of productive systems.

Introduction

Maintenance is used here to refer to actions taken to increase the overall availability of equipment in ageing productive systems. Maintenance actions such as repair, cleaning, testing, lubrication, and replacement are performed to improve mechanical integrity, aid in preventing process safety incidents, and avoid costly downtime due to unplanned shutdowns and slowdowns. Maintenance involves optimal resource allocation and the decisions of when, where and how to do maintenance are key.

Commonly adopted maintenance policies in industry include: (i) basing decisions on mean time to failure (MTTF) recommendations from original equipment manufacturers (OEM), (ii) scheduling maintenance at fixed intervals based on internal company data, (iii) corrective maintenance in which selected equipment are run to failure, (iv) condition-based monitoring and predictive maintenance, (v) risk-based inspection, and (vi) reliability-centered preventive maintenance.

Selection of the appropriate maintenance policy is in part informed by data availability, company culture, and the level of expertise available to create and provide support for developed solutions. It is noted here that the time horizon over which maintenance decisions are made influences maintenance policy selection. Planning is used here to denote high-level decisions taken over months or years and is distinguished from scheduling in which decisions are taken over hours, days, and weeks. A generic example of a maintenance plan is provided in Table 1 in which different equipment are tested (T) and replaced (R) over a five-year planning horizon.

Table 1. Generic maintenance plan

Equipment	Y1		Y2		Y3		Y4		Y5	
V-001		T		T		T		T		T
V-002	T		R		T		T		R	
⋮										
P-27	T	T	T	T	T	R	T	T	T	T
⋮										
C-11	T		T		R		T		T	

Regardless of the maintenance policy selected, certain factors affect optimal resource allocation:

1. Company resources are limited and need to be carefully allocated among operations; business improvement projects; and health, safety and environmental (HSE) projects. The portion of the budget allocated to maintenance is consequently finite and must be decided *a priori*.
2. There are monetary costs associated with maintenance actions and increasing maintenance expenditure leads to diminishing marginal gains in reliability. In other words, the objectives of minimizing cost and maximizing reliability are conflicting and maintenance planning is multi-objective in nature.
3. Productive systems are often composed of multiple degrading equipment whose complicated interactions impart system complexity.
4. Maintenance actions may be imperfect and do not necessarily restore equipment to either an ‘as good as new’ or an ‘as bad as old’ condition.
5. The functions used to rigorously estimate equipment and system reliability are nonlinear and their parameters are not known with absolute certainty.

It is the simultaneous consideration of these factors that distinguishes this research from other efforts in the academic literature on preventive maintenance. The interested reader is directed to a selected subset of the academic literature for context and background information [1-9].

This research employs techniques from the fields of stochastic programming (SP), multi-objective optimization (MOO), and mixed-integer nonlinear mathematical programming (MINLP) to formulate and solve a constrained maintenance planning model. The objectives considered here are cost minimization and reliability maximization. The key decision variables include: the expected number of repairs, the expected number of replacements, whether or not to do maintenance in a time interval, and the sequence of maintenance actions over the time horizon. The results of the research include an optimal maintenance planning framework, plots of equipment and system reliability over time, the expected maintenance budget, the number of spare parts to keep in inventory, and a Pareto front of optimal maintenance plans corresponding to different system reliability thresholds.

This paper proceeds with a description of the methodology used and formulates the maintenance planning model. A case study and preliminary results are subsequently presented to illustrate the multi-objective stochastic optimal maintenance planning approach.

Methodology

The maintenance model used here is data-driven and as such the first step of the methodology is obtaining equipment failure data. In the absence of detailed and complete maintenance and failure records, Monte Carlo methods are used to simulate the equipment failure data based on expert judgement and summary statistics such as the mean time to failure (MTTF). After data validation, maximum likelihood estimation (MLE), or as recourse, expert judgement, is used to determine the values of the reliability model parameters and their uncertainties.

Following estimation of the reliability model parameters, the optimization model is formulated. This formulated model includes the various considerations mentioned above and is a multi-period multi-objective stochastic mixed-integer nonlinear mathematical programming model. Optimization is then performed and an iterative procedure of examining and validating the results is carried out until they are deemed satisfactory. The final step of the methodology is visualization and assessment of the results. The present methodology has been formalized into the optimal maintenance planning framework shown in Figure 1.

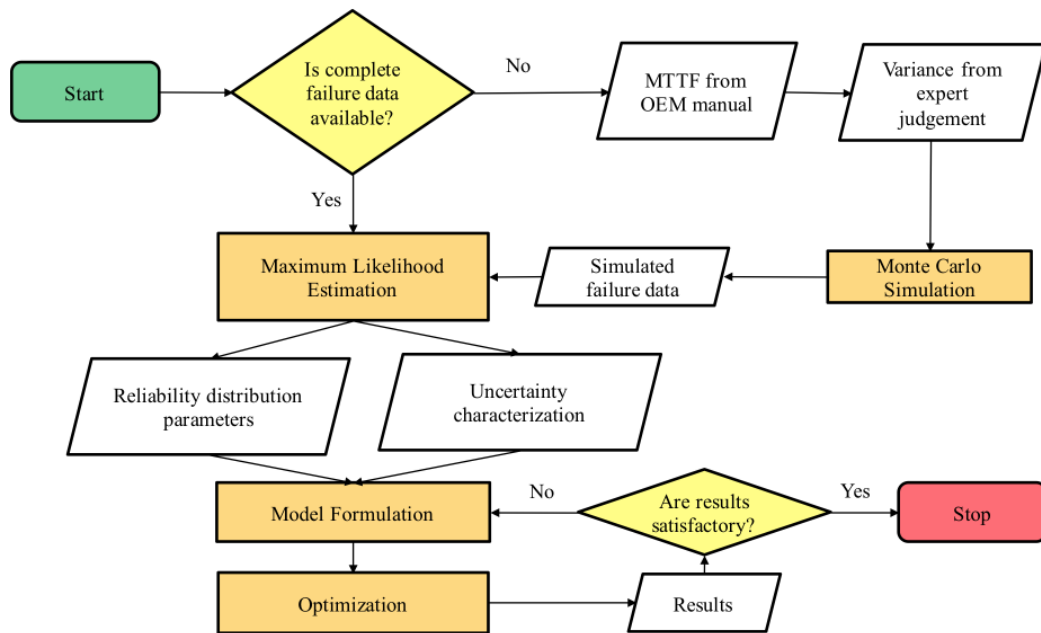


Figure 1. Optimal maintenance planning framework

Model formulation

Equipment are indexed by $i = 1, 2, \dots, I$. Maintenance actions are indexed by $k = 1, 2, \dots, K$, where k_1 is used to denote the absence of a maintenance action, k_2 is used to denote repair, and k_3 is used to denote replacement. Time intervals are indexed by $t = 1, 2, \dots, T$.

The decision variables used in the model are the Weibull scale parameter, τ , the effective age, age, the number replacements, n_{spares} , the number of repairs, n_{repairs} , and whether or not to perform a maintenance action in a time interval, m . These variables are bounded by (1-5).

$$\text{age}^L \leq \text{age}(i, t) \leq \text{age}^U \quad \forall i \in I, t \in T \quad (1)$$

$$0 \leq \tau_E(i) \leq \tau^U \quad \forall i \in I \quad (2)$$

$$n_{\text{spares}}(i) \leq n_{\text{spares}}^U \quad \forall i \in I \quad (3)$$

$$n_{\text{repairs}}(i) \leq n_{\text{repairs}}^U \quad \forall i \in I \quad (4)$$

$$m(i, k, t) = \{0, 1\} \quad \forall i \in I, k \in K, t \in T \quad (5)$$

The first objective, (J1), is minimization of cost, Z . The cost considered here is a function of the maintenance actions, and is parameterized by the cost coefficients C_k and c_{spares} . The second objective, (J2), is the implicit maximization of system reliability, R_{sys} .

$$\min Z = \sum_i^I \sum_k^K \sum_t^T C_k m(i, k, t) + \sum_i^I \sum_t^T c_{\text{spares}} m(i, k_3, t) \quad (J1)$$

$$\ln R_{\text{sys}}(t) \geq \ln \tilde{R}(e) \quad \forall t \in T, e \in \xi \quad (J2)$$

The multi-objective method used here is the ε -constraint method with predefined reliability thresholds, $\tilde{R}(e)$ where $e = 1, 2, \dots, \xi$. A logarithmic transformation has been used here for consistency with other logarithmic transformations used in the model to reduce nonlinearity. It is noted as that (J2) is linear and convex. These objectives are optimized subject to scalar and vector equality and inequality constraints (6-18).

Constraint (6) enforces the performance of at most one type of maintenance action in each time interval.

$$\sum_k^K m(i, k, t) \leq 1 \quad \forall i \in I, t \in T \quad (6)$$

Constraints (7) and (8) determine the expected cumulative number of repairs, and expected number of replacements performed over the time horizon respectively.

$$n_{\text{repairs}}(i, t) = \sum_{t'}^t m(i, k_2, t') \quad \forall i \in I, t \in T \quad (7)$$

$$n_{\text{spares}}(i) = \sum_t^T m(i, k_3, t) \quad \forall i \in I \quad (8)$$

An effective age model is used to capture the effect of the different maintenance actions on equipment condition and is shown in constraints (9) and (11-14). The increase in equipment age in the absence of maintenance is denoted t_d and is equivalent to the time discretization used. The imperfect maintenance factor, α_k , is used to account for maintenance actions that restore equipment to a condition between as-good-as new (AGAN) and as-bad-as-old (ABAO). It is noted here that the use of the effective age model in the context of optimization results in a nonconvex bilinear-integer-continuous (BIC) term shown in (10) which has been reformulated and replaced by (11-14).

$$\text{age}(i, t) = [\text{age}(i, t - 1) + t_d] - \sum_k^K \text{BIC}(i, k, t) \quad \forall i \in I, t \in T \quad (9)$$

$$\text{BIC}(i, k, t) = m(i, k, t)(1 - \alpha_k)[\text{age}(i, t - 1) + t_d] \quad \forall i \in I, k \in K, t \in T \quad (10)$$

$$m(i, k, t)\text{age}^L \leq \text{BIC}(i, k, t) \quad \forall i \in I, k \in K, t \in T \quad (11)$$

$$\text{BIC}(i, k, t) \leq m(i, k, t)\text{age}^U \quad \forall i \in I, k \in K, t \in T \quad (12)$$

$$[(1 - \alpha_k)(\text{age}(i, t) + t_d)] - (1 - m(i, k, t))\text{age}^U \leq \text{BIC}(i, k, t) \quad \forall i \in I, k \in K, t \in T \quad (13)$$

$$\text{BIC}(i, k, t) \leq [(1 - \alpha_k)(\text{age}(i, t) + t_d)] - (1 - m(i, k, t))\text{age}^L \quad \forall i \in I, k \in K, t \in T \quad (14)$$

The scale parameter is described here using a triangular distribution from which a corresponding discrete distribution is constructed. This discrete distribution consists of scale parameter realizations, τ , and realization probabilities, p , for different scenarios $\zeta = 1, 2, \dots, Z$. It has been assumed here that the number of repairs to date increases the lifetime of process equipment and that this corresponds mathematically to an increase in the probability of realizing higher-magnitude scale parameter scenarios. The constraints used to model the scale parameter and its decision-dependent uncertainty are summarized in (15) and (16).

$$p(\zeta, t) = f(n_{\text{repairs}}(i, t)) \quad \forall i \in I, t \in T \quad (15)$$

$$\tau_E(i, t) = \sum_{\zeta}^Z p(\zeta, t)\tau(\zeta) \quad \forall i \in I, t \in T, \zeta \in Z \quad (16)$$

A nonlinear Weibull model is used to describe the equipment reliability in (17) based upon the calculated effective age, the shape parameter, β , and the expected scale parameter, τ_E . This is followed finally by calculation of the system reliability in (18). It is noted that (18) has formulated for a series system, reformulated to reduce computational intractability, and can be adapted to other system configurations.

$$\ln(R(i, t)) = \left(\frac{1}{\tau_E(i)^{\beta(i)}} \right) \text{age}(i, t)^{\beta(i)} \quad \forall i \in I, t \in T \quad (17)$$

$$\ln R_{\text{sys}}(t) = \sum_i^I \ln R(i, t) \quad \forall t \in T \quad (18)$$

The formulated model consists of (J1), (J2) and constraints (1-5), (6-9), (11-14), and (15-18).

Case study description

The system considered for the case study is presented in Figure 2 and consists of three identical centrifugal pumps in series. The parameters used for the case study are shown in Table 2.

Table 2. Model parameter values

Parameter		Value(s)
Equipment age lower bound, yr	age^L	0
Equipment age upper bound, yr	age^U	5
Time interval, yr	t_d	0.5
Scale parameter scenarios, yr	$\tau(\zeta)$	2.6, 3.0, 3.4
Scale parameter upper bound, yr	τ^U	3.6
Number of replacements upper bound	n_{spares}^U	10
Number of repairs upper bound	n_{repairs}^U	10
Normalized cost per repair	C_k	1
Normalized cost per replacement	C_{spares}	10
System reliability thresholds	\widetilde{R}_e	0.9, 0.95, 0.99, 0.995, 0.999
Imperfect maintenance factors	α_k	1, 0.1, 0
Shape parameter	$\beta(i)$	1.5, 1.5, 1.5

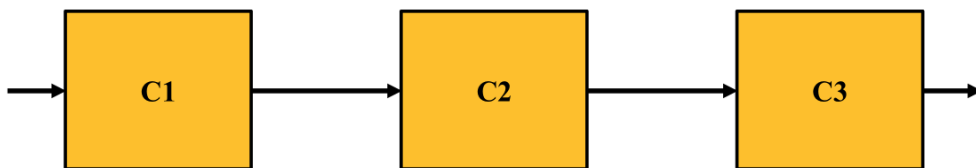


Figure 2. System

Results and discussion

Two sets of preliminary optimization results are provided here. The results consist of maintenance plans showing the optimal sequence of repairs (I) and replacements (P), plots of the corresponding equipment and system reliability against time, and a sensitivity analysis in the form of a Pareto front.

The first set of results corresponds to a maintenance policy in which a recommendation of a manufacturer to repair equipment once every three years is followed. The optimal maintenance plan corresponding to this policy is shown in Table 3 and was produced by adapting (7) and (8). It was observed that this policy rendered the model infeasible until the system reliability thresholds (J2) were relaxed. In less mathematical terms, this maintenance policy was inconsistent with the goal of maintaining system reliability above set thresholds over the entire time horizon. This is visualized in Figure 3, from which it can be observed that the equipment and system reliability profiles are below 90%, and by extension 99.5%, over the majority of the time horizon.

Table 3. Maintenance plan based on a manufacturer recommendation

Equipment	n _{spares}	Y1		Y2		Y3		Y4		Y5	
C1	0					I					
C2	0						I				
C3	0						I				

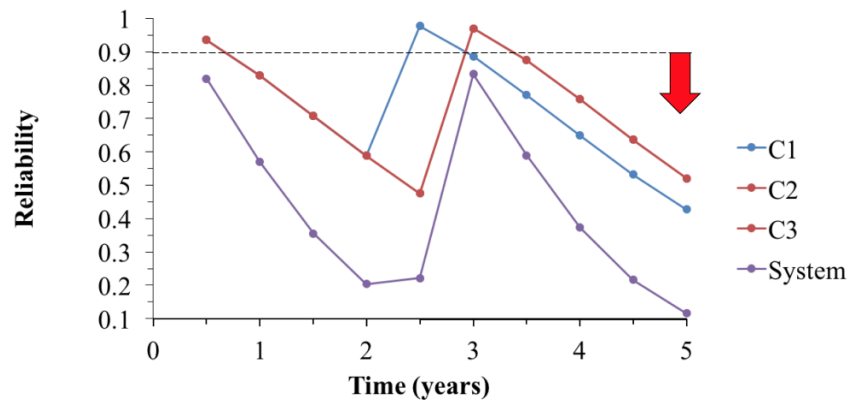


Figure 3. Equipment and system reliability based on a manufacturer recommendation

The second set of results corresponds to maintenance performed according to the methodology presented in this paper. The maintenance plan corresponding to a system reliability threshold of 99.5% is presented in Table 4.

Table 4. Maintenance plan based on present methodology

Equipment	n _{spares}	Y1		Y2		Y3		Y4		Y5	
C1	5	P	I	I	I	I	P	P	P	I	P
C2	4	I	I	P	P	P	I	I	I	P	I
C3	1	I	P	I	I	I	I	I	I	I	I

The equipment and system reliability profiles are visualized in Figure 4 and system reliability is seen to be maintained above the set threshold over the entire time horizon.

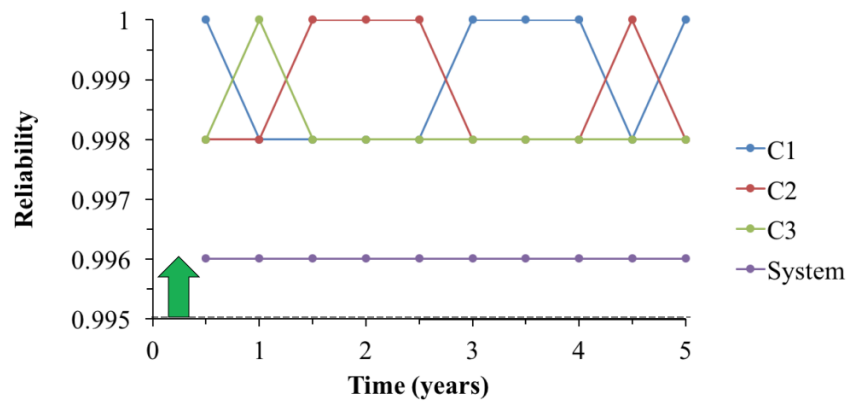


Figure 4. Equipment and system reliability based on the present methodology

Finally, it is noted that the results are dependent on the parameters used. The effect of changing the system reliability threshold is shown in Figure 5 in which each point corresponds to a different optimal maintenance plan and a trade-off between cost and reliability is observed.

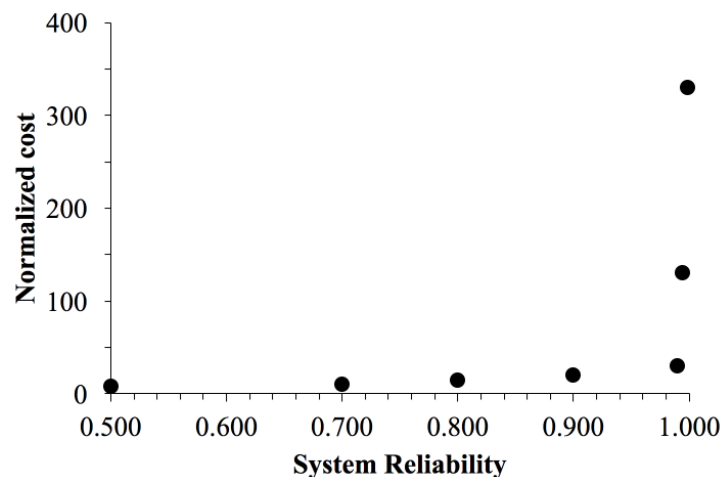


Figure 5. Pareto front of optimal maintenance plans

Conclusion

Maintenance planning is complicated by resource scarcity, system complexity, reliability nonlinearity and parametric uncertainty. This paper presents a maintenance optimization framework and employs multi-objective optimization under uncertainty to help guide resource allocation. Preliminary results show that application of the techniques adopted in this paper can result in improvements in equipment and system reliability as compared to implementation of a manufacturer recommendation.

The optimal maintenance planning framework and formulated model can be adapted to ageing productive systems in different industries inclusive of refining, chemical production, and manufacturing both onshore as well as offshore.

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